the corresponding digit in the expression of b in the binary scale is 1, giving the term of value $1 \times 2^{r-1}$ in the development of b in powers of 2. If, on the other hand, the number in the rth line of the second column is even there will be no remainder, and the corresponding digit in the binary scale expression of b is 0, giving the term of value $0 \times 2^{r-1}$ in the development of b. But the number in the rth line of the first column is $2^{r-1} \times a$, and since all numbers in the first column that are opposite even numbers in the second column are stricken out, the sum of the remaining numbers will be precisely $b \times a$.

A numerical example will make this much clearer. The work for the multiplication of 14 and 83 would appear (except for the figures in parentheses) as follows:

$(2^0 \times 14 =)$* 14	83
$(2^1 \times 14 =)$* 28	41
$(2^2 \times 14 =)$ 56	20
$(2^3 \times 14 =)$ 112	10
$(2^4 \times 14 =)$*224	5
$(2^5 \times 14 =)$ 448	2
$(2^6 \times 14 =)$*896	1
1162	

The number 83 expressed in the binary scale of notation would be 1010011 (*i.e.*, $83 = 2^1 + 2^4 + 2^1 + 2^0$). Thus the sum of those parentheses marked with an asterisk is $(2^6 + 2^4 + 2^6 + 2^0) \times 14$, or 83×14.

In presenting a similar discussion Professor U. G. MITCHELL cited the article in this MONTHLY, *1918*, 139–142, by Professor R. C. ARCHIBALD, entitled: "The binary scale of notation, a Russian peasant method of multiplication, the game of nim, and Cardan's rings." Many references are there given to the literature of the history and discussion of the binary scale and its applications.—EDITORS.

Also solved by T. M. BLAKSLEE, B. A. BERNSTEIN, PAUL CAPRON, CARL GUNDERSEN, W. H. HAYS, A. M. KENYON, THEODORE LINQDUIST, ROSCO LAMONT, H. F. MACNEISH, L. C. MATHEWSON, H. L. OLSON, ARTHUR PELLETIER, W. B. PIERCE, D. H. RICHERT, H. S. UHLER, and C. C. WYLIE.

2822 [1920, 185]. Proposed by A. M. HARDING, University of Arkansas.

Show that the sum of the series:

$$1 \cdot + 3\cdot 2 + 5\cdot 2^2 + 7\cdot 2^3 + \cdots + (2n-1)2^{n-1}$$

(to n terms) is $3 - 2^n + (n-1)2^{n+1}$.

SOLUTION BY LOUIS O'SHAUGHNESSY, Virginia Polytechnic Institute.

Set

$$S = 1 + 3\cdot 2 + 5\cdot 2^2 + 7\cdot 2^3 + \cdots + (2n-3)2^{n-2} + (2n-1)2^{n-1}.$$

Then

$$2S = 2 + 3\cdot 2^2 + 5\cdot 2^3 + \cdots + (2n-5)2^{n-2} + (2n-3)2^{n-1} + (2n-1)2^n.$$

Hence,

$$S = -1 - 2\cdot 2 - 2\cdot 2^2 - 2\cdot 2^3 - \cdots - 2\cdot 2^{n-2} - 2\cdot 2^{n-1} + (2n-1)2^n,$$

$$= -1 - \sum_2^n 2^n + (2n-1)2^n = -1 - (2^{n+1} - 4) + (2n-1)2^n = 3 - 2^n + (n-1)2^{n+1}.$$

Also solved by T. M. BLAKSLEE, H. N. CARLETON, P. J. DA CUNHA, E. B. ESCOTT, R. M. GINNINGS, H. HALPERIN, HARRY LEVY, L. C. MATHEWS, H. L. OLSON, ARTHUR PELLETIER, A. V. RICHARDSON, ETHELDRED A. WILLMOTT, and C. C. WYLIE.